Profondità critica.

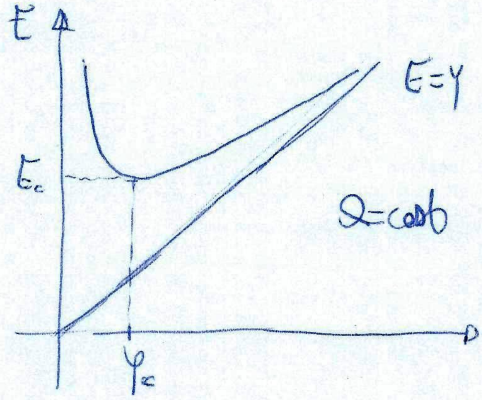
Carico specifico: $E = H - \eta = h + \frac{\alpha V^2}{2g} - \eta = \cancel{\eta} + \cancel{\eta} + \frac{\alpha V^2}{2g} - \eta =$

$$= Y + \frac{\alpha V^2}{2g}$$

Noto che $\alpha \cong 1$!

$$E = Y + \frac{V^2}{2g}$$

Graficamente:



$\lim_{Y \to +\infty} E = +\infty \qquad \lim_{Y \to 0} E = +\infty$

Stato critico per Y_c:

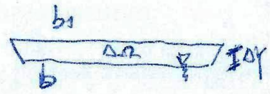
$$\frac{dE}{dY} = 1 + \frac{\partial}{\partial Y}\left(\frac{Q^2}{2g\Omega^2}\right) = 1 +$$

$$+ \frac{Q^2}{2g} \cdot \frac{\partial}{\partial Y}\left(\frac{1}{\Omega^2}\right) =$$

$$= 1 + \frac{Q^2}{2g}\left(-\frac{2}{\Omega^3} \cdot \frac{\partial \Omega}{\partial Y}\right) =$$

$$= 1 - \frac{Q^2}{g\Omega^3} \cdot \frac{\partial \Omega}{\partial Y}$$

Abbiamo:



$$\frac{\partial \Omega}{\partial Y} = \lim_{\Delta Y \to 0} \frac{\Delta \Omega}{\Delta Y} = \lim_{\Delta Y \to 0} \frac{(b + b_1)\,\cancel{\Delta Y}/2}{\cancel{\Delta Y}} =$$

$$= \lim_{\Delta Y \to 0} \frac{b + b_1}{2} = \frac{2b}{2} = b$$

Quindi $\frac{dE}{dY} = 1 - \frac{Q^2}{g\Omega^3} \cdot b = 0 \Rightarrow \frac{Q^2}{g\Omega^3} b = 1 \Rightarrow \frac{Q^2}{g} = \frac{\Omega^3}{b}$

La profondità critica soddisfa allora l'equazione:

$$\frac{Q^2}{g} = \left[\frac{\Omega^3}{b}\right]_{Y = Y_c}$$

Per alveo rettangolare:

$$\frac{Q^2}{g} = \frac{b^{\cancel{3}\,2} Y_c^3}{\cancel{b}} = b^2 Y_c^3 \Rightarrow Y_c = \left(\frac{Q^2}{b^2 g}\right)^{1/3}$$

Per cui E_c è:

$$E_c = Y_c + \left[\frac{Q^2}{2g\Omega^2}\right]_{Y_c} = \left[Y + \frac{\Omega^{\cancel{3}\,1} \cancel{g}}{2\cancel{g} b \cancel{\Omega^2}}\right]_{Y = Y_c} = \left[Y + \frac{\Omega}{2b}\right]_{Y = Y_c}$$

Profondità critica:

$$\frac{Q^2}{g} = \left[\frac{\Omega^3}{b}\right]_{Y=Y_c}, \quad \text{per alveo rettangolare } \Omega = b \cdot Y \Rightarrow \frac{Q^2}{g} = \frac{b^{\cancel{3}2} Y_c^3}{\cancel{b}} = b^2 Y_c^3$$

$$\Rightarrow Y_c = \sqrt[3]{\frac{Q^2}{g\,b_0^2}}$$

Da notare $Y = f(Q, b, k_s, i_f)$, qui $Y_c = f(Q, b)$.

Per alveo trapezio:

$$F(Y) = \frac{Q^2}{g} - \frac{\Omega^3}{b} \Rightarrow F(Y) = \frac{Q^2}{g} - \left(b_0 Y + \frac{Y^2}{tg\alpha}\right)^3 \cdot \frac{1}{b_0 + \frac{2Y}{tg\alpha}}$$

$$F'(Y) = -3\left(b_0 Y + \frac{Y^2}{tg\alpha}\right)^{3-1} \cdot \frac{b_0 + 2\frac{Y}{tg\alpha}}{b_0 + \frac{2Y}{tg\alpha}} + \left(b_0 Y + \frac{Y^2}{tg\alpha}\right)^3 \cdot \left(b_0 + \frac{2Y}{tg\alpha}\right)^{-1-1} \cdot \frac{2}{tg\alpha} =$$

$$= -\frac{\left(b_0 Y + Y^2/tg\alpha\right)^3}{\left(b_0 + \frac{2Y}{tg\alpha}\right)} \left[\frac{3\left(b_0 + 2Y/tg\alpha\right)}{\left(b_0 Y + \frac{Y^2}{tg\alpha}\right)} - \frac{2}{\left(b_0 + \frac{2Y}{tg\alpha}\right) \cdot tg\alpha}\right]$$

Usiamo ancora:

$b_0 = 5\,m \qquad Q = 20\,m^3/s \qquad k_s = 30\,m^{1/3}s^{-1} \qquad \alpha = 45° \qquad i_f = 0{,}0030$

Tabella:

n	Y_n [m]	b [m]	Ω [m²]	B [m]	$F(Y)$	$F'(Y)$	ε
0	1,177	7,354	7,271	8,33	-11,482	-144,361	-
1	1,097	7,196	6,688	8,103	-0,816	-119,039	0,068
2	1,090	7,180	6,638	8,082	0,036	-120,845	0,006
3	~1,090	7,180	6,638	8,082	0,036	-120,845	~0